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PHYSICAL INVENTORY TAKING USING INVENTORY RECORDS AND DIRECT ON--ETC(U)

APR 78 H O HARTLEY, W M BOWEN DAAG29-77-G-0086

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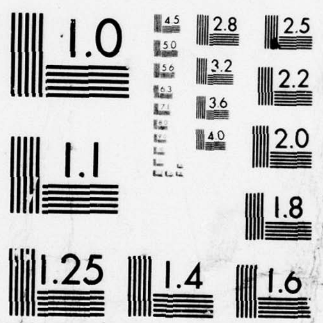

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An integral part of any commercial activity is the periodic counting of assets. In some larger establishments (such as a complex of warehouses and arsenals) this is a major undertaking involving many man-hours of labor. It is customary to take such an inventory on an annual basis. This report provides an approach which employs a sampling design in which items are inspected on a rotating basis but also uses the time series of inventory records. The present procedure, therefore provides (1) a quality control of the inventory records protecting the establishment from

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20. ABSTRACT CONTINUED

having to use inventory records that are seriously in error, and (2) a method of computing best estimates of complete inventories for items at any specified time with computable statistical errors of inventory estimation. As indicated above, the method is a combination of rotation sampling of items and the application of an autoregressive time series analysis applied to the inventory records.

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Technical Report No. 4

PHYSICAL INVENTORY TAKING USING INVENTORY RECORDS AND DIRECT ON-THE-SPOT MONITORING

by

H. O. Hartley and William Michael Bowen

April 1978

1. Introduction

An integral part of any commercial activity is the periodic counting of assets. In some larger establishments (such as a complex of warehouses and arsenals) this is a major undertaking involving many man-hours of labor. It is customary to take such an inventory on an annual basis, particularly for auditing purposes. This procedure has two major disadvantages: (a) Since the inventory will usually comprise a very large number of individual items that have to be counted, an annual inventory will dislocate normal procedures and often require special labor; (b) if no "on-the-spot" inventories are taken for a whole year, the inventory record is liable to get "out-of-date" with a serious accumulation of errors which makes, in certain cases, the inventory records useless for an up-to-date picture on the item counts.

It was with these considerations in mind that M. R. Bryson (1960) suggested a method of taking inventories piecemeal on a sampling basis. The present approach differs from Bryson's method in that it employs a sampling design in which items are inspected on a rotating basis but also uses the time series of inventory records. The present procedure, therefore provides

(1) a quality control of the inventory records protecting the establishment from having to use inventory records that are seriously in error, and

(2) a method of computing best estimates of complete inventories for items at any specified time with computable statistical errors of inventory estimation.

As indicated above, the method is a combination of rotation sampling of items and the application of an autoregressive time series analysis applied to the inventory records.

The method as described in the subsequent sections applies to a situation (often encountered) where storage facilities are at a premium so that large incoming shipments as well as large quantities of stock on hand are to be avoided if possible. Reorder point policies which allow for planned large reorders and large cross leveling actions are briefly discussed in Section 8.

At the present time, the process here described could not be applied to actual inventory data since this would have required the adoption of our technique as an inventory control strategy. However, it is hoped to test the procedure on simulated data in the future before submitting the report for publication.

2. Description of the Problem and Notation

Our notation will be in accordance with Bryson (1960). Within a warehouse, or supply center, there are different types of items being stored. Henceforth, the term "item" will denote a type of item, such as a bolt or a more complex assembly, and the term "piece" will denote an individual unit of the type of item. Thus we are interested in the number of pieces of each item that are on hand at a given time.

A continuous record is kept of all activities and the resulting changes in the amount of each item on hand. Let x_{it} denote this inventory record for item i at time t. The time t may be measured in weeks or months depending on circumstances. If a physical count is made on item i at time t, this actual inventory is denoted y_{it}.

The recording of activities for an item is not instantaneous. In fact, due to delays caused by paperwork and other administrative processing, the inventory record is probably somewhat out of date compared with the actual inventory.

The supply center is to be "stratified" so that items of similar activity and having similar administrative delays of record keeping are placed within the same stratum. We now confine our discussion to activities within a single stratum, noting that the technique to be described applies to all strata on an individual basis.

3. Rotational Design

Let us define a "short time interval" such that the inventory records are examined at the beginning of each such interval. For fast moving items this interval may be a week. For slower moving items it may be a month but should be the same for items in a stratum.

The items within a stratum are randomly divided into rotational groups, such that a group of items is physically inventoried in a particular week (month) and then is not inventoried again until all such rotational groups have been inventoried. Therefore, the desired frequency of a physical inventory of each item must be considered in determining the number and hence the size of rotational groups. If, for example, it is desirable to inventory each item at least annually, there should be no more than 52 (12) rotational groups, perhaps of equal size.

As an illustration consider the rotational monthly inventory plan for twenty-four items over a two year period shown in Figure 1.

We now confine the scope of our discussion to an individual item, noting of course that the discussion applies to all items on an individual basis.

Let the term "period" denote the time interval between successive inventories of an item. Then period p is the time interval which includes the pth inventory of an item and all weeks (months) preceding, but not including, the $(p + 1)^{st}$ inventory of the same item.

4. A Model for the "Normal" Fluctuation in the Inventory Record

The following rationale was used to derive the autoregressive time series model which is described below. It was not considered realistic

Figure 1

	J	F	M	A	M	J	J	A	S	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D
1	*												*											
2	*												*											
3		*												*										
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Where '*' indicates the item is being inventoried.

to assume that the inventory records constitute a stationary time series for there may well be both upward and downward linear or higher order trends over the period or (indeed) seasonal fluctuations. However, following Box and Jenkins (1970) it was assumed that a stationary process could be reached if the original time series of inventory records would be differenced. A third order difference was therefore assumed to constitute a stationary process. Such an assumption would allow for linear and parabolic like trends over a period p.

To fix the idea, the third difference in the records for item i at time t in period p is given by

$$z_{ipt} = \Delta^3 x_{ipt} = (x_{ipt} - 3x_{ipt-1} + 3x_{ipt-2} - x_{ipt-3}) \quad . \tag{1}$$

where x_{ipt} denotes the t^{th} record of period p and the third difference z_{ipt} is defined by (1) above. An assumption to be made at this point is that under normal conditions the third differences form a stationary autoregressive process of the first order. However, when an inventory is taken on item i and the record is updated, there may be a sizable jump in x_{ip1} and hence the first third difference assumed to follow the stationary process is z_{ip4}. Therefore, z_{ipt} can be calculated only after the 4th weekly (monthly) record is available for a particular period.

Since our discussion is confined to an individual item, for notational simplicity we drop the item subscript, i, and define z_{pt} to be the third difference at time t within period p. And the number of weeks (months) in the p^{th} period will be denoted N_p.

5. Monitoring

For the purpose of monitoring the monthly record, a time series forecast of z_{pt+1} is obtained from which a conditional confidence interval for x_{pt+1} is constructed. When the next record x_{pt+1} becomes available, it is compared with the confidence interval to determine whether or not further action is necessary. A detailed discussion of this monitoring procedure follows.

A summary flowchart of the procedure to be discussed is given in Figure (A.1) and Figure (A.2) Appendix A, where the stages of the procedure have been numbered in a logical sequence. In what follows we shall frequently make reference to this flowchart.

To fit the first order autoregressive process

$$z_{pt+1} = \phi_p z_{pt} + \varepsilon_{pt+1} , \tag{2}$$

an estimate of ϕ_p is obtained using the method discussed by Box and Jenkins (1970). The computational formula for this estimate, denoted $r_1(p)$, for a completed period is

$$r_1(p) = \frac{\left[\sum_{t=5}^{N_p} (z_{pt-1} - \bar{z}_p)(z_{pt} - \bar{z}_p)/(N_p - 5) \right]}{\left[\sum_{t=4}^{N_p} (z_{pt} - \bar{z}_p)^2/(N_p - 4) \right]} . \tag{3}$$

And for a period not yet completed, let N_p^* denote the number of months for which the inventory records are available. Then substituting N_p^* for N_p in (3) gives the desired result. Note that N_p^* must be greater than five in order to obtain $r_1(p)$.

A forecast of z_{pt+1} is then given by

$$\hat{z}_{pt+1} = r_1(p)\, z_{pt} , \quad t \geq 6 . \tag{4}$$

The corresponding forecast of x_{pt+1} is obtained by substituting the expression for z_{pt} from (1) into (4) above. This yields

$$\hat{x}_{pt+1} = \hat{z}_{pt+1} + 3x_{pt} - 3x_{pt-1} + x_{pt-2} \; . \tag{5}$$

To obtain the conditional confidence interval for x_{pt+1} given x_{pt}, x_{pt-1}, x_{pt-2}, and x_{pt-3} we note that the only variable in (5) is the forecast of z_{pt+1} that is $\hat{z}_{pt+1}$ so that the variance of $\hat{z}_{pt+1}$ given the preceding z_{pt} is also the conditional variance of $\hat{x}_{pt+1}$ given the preceding x_{pt}. To obtain the former we use standard formulas for an autoregressive process. This is given by

$$s_p^2 = (1 - r_1^2(p))\hat{\sigma}_{z_p}^2 \; , \tag{6}$$

where $\hat{\sigma}_{z_p}^2$ is the estimate of $\sigma_{z_p}^2$, the variance of the z_{pt} for period p, having the computational form

$$\hat{\sigma}_{z_p}^2 = \sum_{t=4}^{N_p} (z_{pt} - \bar{z}_p)^2/(N_p - 4) \; .$$

Again we note that $N_p^* > 4$ is substituted for N_p when the period is not complete.

Obtaining $u_{(1-\alpha/2)}$ from the Standard Normal Tables, a $100(1 - \alpha)\%$ confidence interval for x_{pt+1} is given by

$$\hat{x}_{pt+1} \pm \sqrt{s_p^2}\, u_{(1-\alpha/2)} \; . \tag{7}$$

We note at this point that during the early weeks (months) of a period, the number of third differences may be insufficient to obtain "reliable" estimates of ϕ_p and $\sigma_{z_p}^2$. In fact, when months are used as time intervals, the number of months in an entire period may be considered insufficient. For this reason it is suggested that pooled estimates of the overall ϕ and σ_z^2 be used for the purpose of constructing confidence intervals for x_{pt+1} during the early weeks (months) of a period. If it can be established that ϕ_p and $\sigma_{z_p}^2$ are constant for all periods, the pooled estimates might be preferred over $r_1(p)$ and $\hat{\sigma}_{z_p}^2$ under any circumstances. However, a word of caution is in order concerning the use of the pooled estimates for other than the early weeks (months) of a period, especially when ϕ_p and $\sigma_{z_p}^2$ are not constant over all periods. Bias may be introduced through the pooled estimates, which may have such an adverse affect on the forecast that the whole purpose for pooling is lost.

The pooled estimate of ϕ would consist of a weighted average of the $r_1(p)$'s over all periods. Specifically,

$$\bar{r}_1 = \frac{\sum_p (N_p - 4) r_1(p)}{\sum_p (N_p - 4)} , \tag{8}$$

where $r_1(p)$ for the current period is not available unless $N_p^* > 5$.

Thus using $\bar{r}_1$ allows forecasting of z_{pt+1} in the current period as soon as the fourth record is available. Whereas using $r_1(p)$ would require that at least the first six records be available.

Letting $\bar{r}_1$ replace $r_1(p)$ in (4), the forecast of z_{pt+1} becomes

$$\tilde{z}_{pt+1} = \bar{r}_1 z_{pt} , \quad t \geq 4 . \tag{9}$$

The resulting forecast of x_{pt+1} is

$$\tilde{x}_{pt+1} = \tilde{z}_{pt+1} + 3x_{pt} - 3x_{pt-1} + x_{pt-2} .$$

The estimate of the conditional variance of $\tilde{x}_{pt+1}$ given the preceding x_{pt} is given by

$$\tilde{s}^2 = (1 - \bar{r}_1^2)\tilde{\sigma}_z^2 ,$$

where $\tilde{\sigma}_z^2$ is the pooled estimate of the variance of z_{pt} over all periods and is given by

$$\tilde{\sigma}_z^2 = \frac{[\sum_p (N_p - 4)\hat{\sigma}_{z_p}^2]}{[\sum_p (N_p - 4)]} ,$$

where again N_p^* replaces N_p for the current period.

The $100(1 - \alpha)\%$ confidence interval for x_{pt+1} is now

$$\tilde{x}_{pt+1} \pm \sqrt{\tilde{s}^2}\, u_{(1-\alpha/2)} , \tag{11}$$

where $u_{(1-\alpha/2)}$ *is obtained from the* Standard Normal Tables, as before.

Once a confidence interval for x_{pt+1} has been constructed, using either (7) or (11) above, the monitoring test is carried out by comparing the observed x_{pt+1} with the confidence interval. This can begin as early as the fifth month of the current period. This is step 2 of Figure (A.1).

If x_{pt+1} falls outside the confidence interval it is an indication that either

(a) there is a sizable error in the last record x_{pt+1},

or

(b) an unexpectedly large transaction has taken place during the last interval.

When investigating the cause of the discrepancy, (a) should be considered first. If an error is found in the records, it can be corrected. The monitoring test is then repeated using the corrected records. If x_{pt+1}

is still outside the confidence interval after correction, or if no error was found in the records, (b) is considered. This requires that an inventory be taken to verify this abnormally large change in the record. This corresponds to steps 3 and 4 of Figure (A.1).

When an inventory occurs due to monitoring, the item is automatically assigned to the rotational group for which the regular inventory is being taken during the present month. Thus an inventory for the item is not taken again until its newly assigned rotational group is scheduled again, unless otherwise dictated by future monitoring. This is step 5 in Figure (A.1).

6. The Estimates of Item-Inventories Under 'Stationary Conditions'

It is usual practice to use x_{ipt} the last inventory-record of each item available at time t as the best source of information concerning the inventory of item i in question. However, it is reasonable to ask whether a better "update" of the inventory could be estimated from the data. This is particularly relevant if x_{ipt} is known to be "out of date" owing to unavoidable delays in updating the records.

The procedure developed below assumes that the third differences z_{ipt} of the inventory record series x_{ipt} follows the postulated stationary process. The monitoring of this process as described in Section 5 has therefore a double purpose, namely (a) to invoke the taking of an actual inventory y_{ipt} if a departure from the stationary process is discovered and (b) the prediction of y_{ipt} if the stationary process cannot be rejected. It is this latter use which we describe in this section which is step 6 of Figure (A.1).

Figure 2 illustrates the method

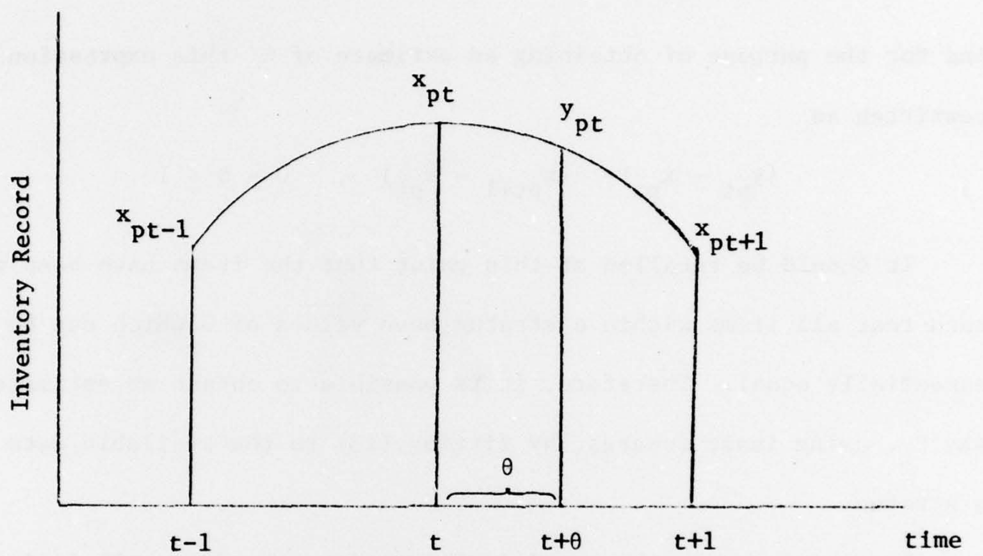

Figure 2

In Figure 2, the delay in the inventory records is denoted θ, where $0 \leq \theta \leq 1$. However, when weeks are used as intervals, it seems possible that the delay could exceed one week. In which case the upper limit of the restriction on θ may be relaxed with caution but should not be allowed to exceed two weeks. If in fact the delay is more than 1.5 weeks, it may be more appropriate to use months as time intervals in the future. Thus the actual inventory at time t, y_{pt}, can be written in terms of θ and the inventory records using either linear or quadratic interpolation. We now consider the estimation of θ and the resulting forecast of y_{pt} for both cases.

The linear interpolate is of course the simpler of the two. It is given by

$$y_{pt} = (1 - \theta)x_{pt} + \theta x_{pt+1} \quad , \quad 0 \leq \theta \leq 1 \ . \tag{12}$$

And for the purpose of obtaining an estimate of θ, this expression can be rewritten as

$$(y_{pt} - x_{pt}) = \theta(x_{pt+1} - x_{pt}) \quad , \quad 0 \leq \theta \leq 1 \ . \tag{13}$$

It should be recalled at this point that the items have been stratified such that all items within a stratum have values of θ which can be assumed essentially equal. Therefore, it is possible to obtain an estimate of θ, say $\hat{\theta}_L$, using least squares, by fitting (13) to the available data within a stratum.

Returning now to the triple subscripting, the data will include y_{ip1} and the corresponding records, x_{ip1} and x_{ip2}, for all i and p, except where the inventory was taken due to monitoring. Note that since a forecast of x_{ipt+1} cannot be obtained until t = 4, the record x_{ip2} must be available before y_{ip1} can be included in the data.

After computing $\hat{\theta}_L$, the restriction $0 \leq \hat{\theta}_L \leq 1$ is imposed. When months are being used as intervals, if $\hat{\theta}_L < 0$, use $\hat{\theta}_L = 0$; or if $\hat{\theta}_L > 1$, use $\hat{\theta}_L = 1$. As mentioned previously, the upper limit may be somewhat relaxed when weeks are used as intervals. Thus, in the case where weeks are used, if $\hat{\theta}_L < 0$ use $\hat{\theta}_L = 0$, but if $\hat{\theta}_L > 1$, $\hat{\theta}_L$ may either be used with caution or set equal to unity. This will obviously give the restricted least squares estimate of θ.

The forecast of the current inventory is then obtained by substituting $\hat{\theta}_L$ for θ and $\hat{x}_{ipt+1}$ for x_{ipt+1} in (12). This gives

$$\hat{y}_{ipt} = (1 - \hat{\theta}_L)x_{ipt} + \hat{\theta}_L \hat{x}_{ipt+1} \tag{14}$$

where $\hat{x}_{ipt+1}$ is the forecast of x_{ipt+1} from either (5) or (10).

When the time interval is a month, it may be more appropriate to use a quadratic interpolation formula. The quadratic interpolate is given by

$$y_{ipt} = \tfrac{1}{2}\theta(1 - \theta)x_{ipt-1} + (1 - \theta^2)x_{ipt} + \tfrac{1}{2}\theta(1 + \theta)x_{ipt+1}$$

$$0 \le \theta \le 1 \ . \tag{15}$$

The estimate of θ, say $\hat{\theta}_Q$, is obtained from the available data in a stratum as follows.

Define

$$Q(\theta) = \sum_{ip} [y_{(i,p,1)} - \tfrac{1}{2}\theta(1 - \theta)x_{(i,p-1,N_{p-1})}$$

$$- (1 - \theta^2)x_{(ip1)} - \tfrac{1}{2}\theta(1 + \theta)x_{(ip2)}]^2 \ . \tag{16}$$

$Q(\theta)$ is tabulated as a function of θ (say at interval $\Delta\theta = .05$), for $0 \le \theta \le 1.2$. The absolute minimum is then determined numerically, using Mathematical Programming by Scanning. The value of θ corresponding to this absolute minimum is then the desired estimate, $\hat{\theta}_Q$.

The forecast of the current inventory is then obtained by substituting $\hat{\theta}_Q$ for θ and $\hat{x}_{ipt+1}$ for x_{ipt+1} in (15). This yields

$$\hat{y}_{ipt} = \tfrac{1}{2}\hat{\theta}_Q(1 - \hat{\theta}_Q)x_{ipt-1} + (1 - \hat{\theta}_Q^2)x_{ipt} + \tfrac{1}{2}\hat{\theta}_Q(1 + \hat{\theta}_Q)\hat{x}_{ipt+1} \tag{17}$$

where $\hat{x}_{ipt+1}$ is the forecast of x_{ipt+1} from either (5) or (10).

Since new data will become available each week (month), the estimate of θ, either $\hat{\theta}_L$ or $\hat{\theta}_Q$, should be updated frequently.

7. A Quality Control Chart Monitoring Regular Inventories Against Inventory Records

When the p^{th} inventory is taken on item i, a new period p begins with y_{ip1} as the updated value of the inventory record. (Step 1, Figure (A.2).) A discrepancy between y_{ip1} and x_{ip1} is expected due to the delay in the records. However, if this discrepancy is unusually large, it may be an indication that either

(a) errors exist in the records which may have accumulated during the previous period

or in the case where y_{ip1} is extremely smaller than x_{ip1},

(b) pilfering or other unexplained losses have occurred.

In order to monitor such discrepancies, a control chart technique can be implemented. Define

$$d_{ip} = y_{ip1} - x_{ip1} ,$$

where only the regular inventories (not resulting from the monitoring discussed in Section 5) are included. Recalling again that the items have been stratified according to activity and delay, an estimate of the variance of the discrepancies is given by

$$s_d^2 = \sum_{ip}\sum d_{ip}^2/n ,$$

where n is the total number of d_{ip}'s available in the stratum. $100(1 - \alpha)\%$ control limits are then given by

$$\pm \sqrt{s_d^2}\ u_{(1-\alpha/2)} . \tag{18}$$

where $u_{(1-\alpha/2)}$ is from the Standard Normal Tables. (Step 2, Figure (A.2).)

If d_{ip} falls outside the control limits, (a) is considered first. If an error is found in the record, it can be corrected. If after correcting the records d_{ip} falls below the lower control limit, or if no error is found and d_{ip} is below the lower limit (b) is considered. In this case appropriate action should be taken to prevent further unexplained losses. (Steps 3, 4, and 5 of Figure (A.2).)

APPENDIX A

Figure (A.1)

Summary Flowchart

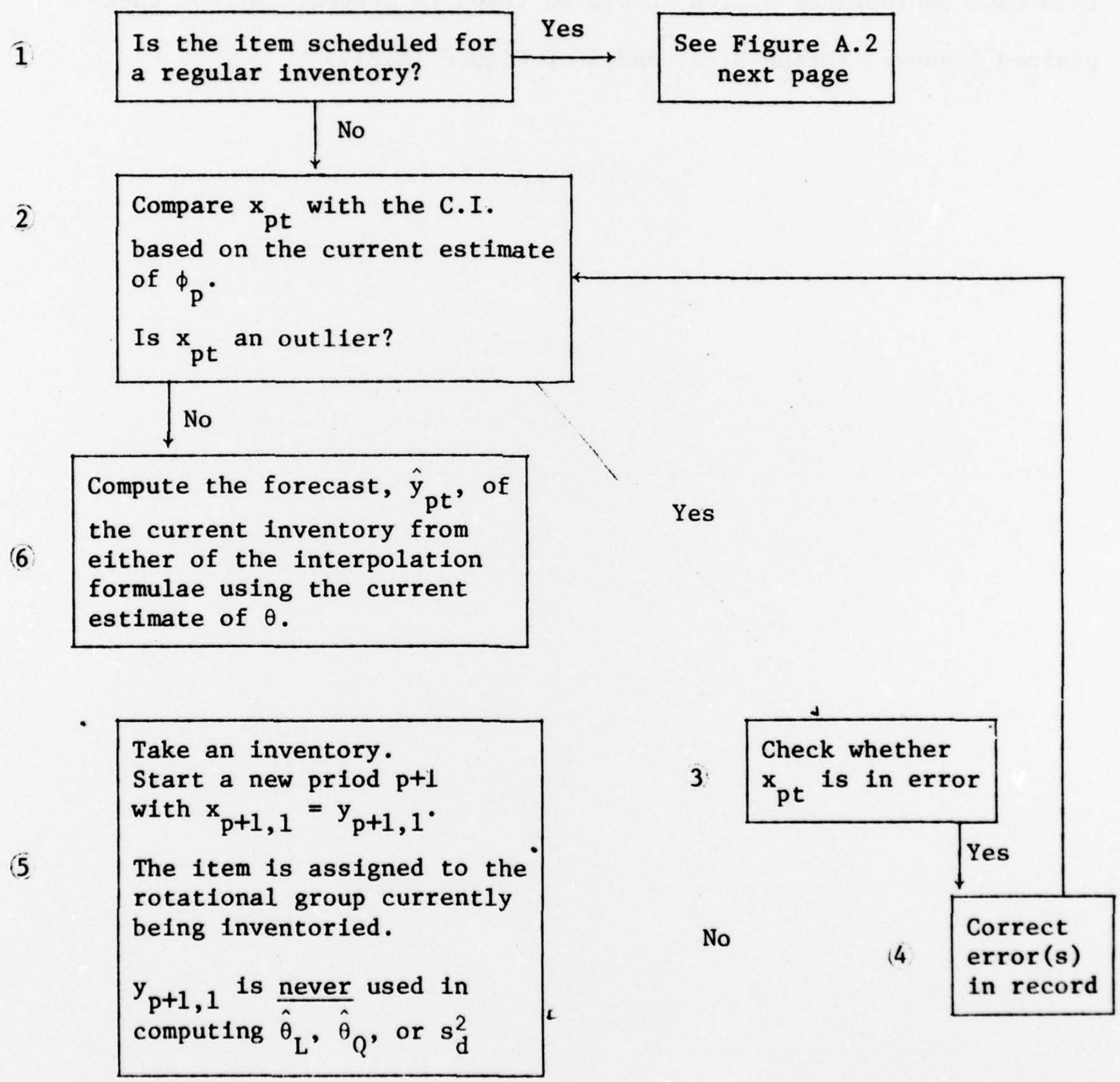

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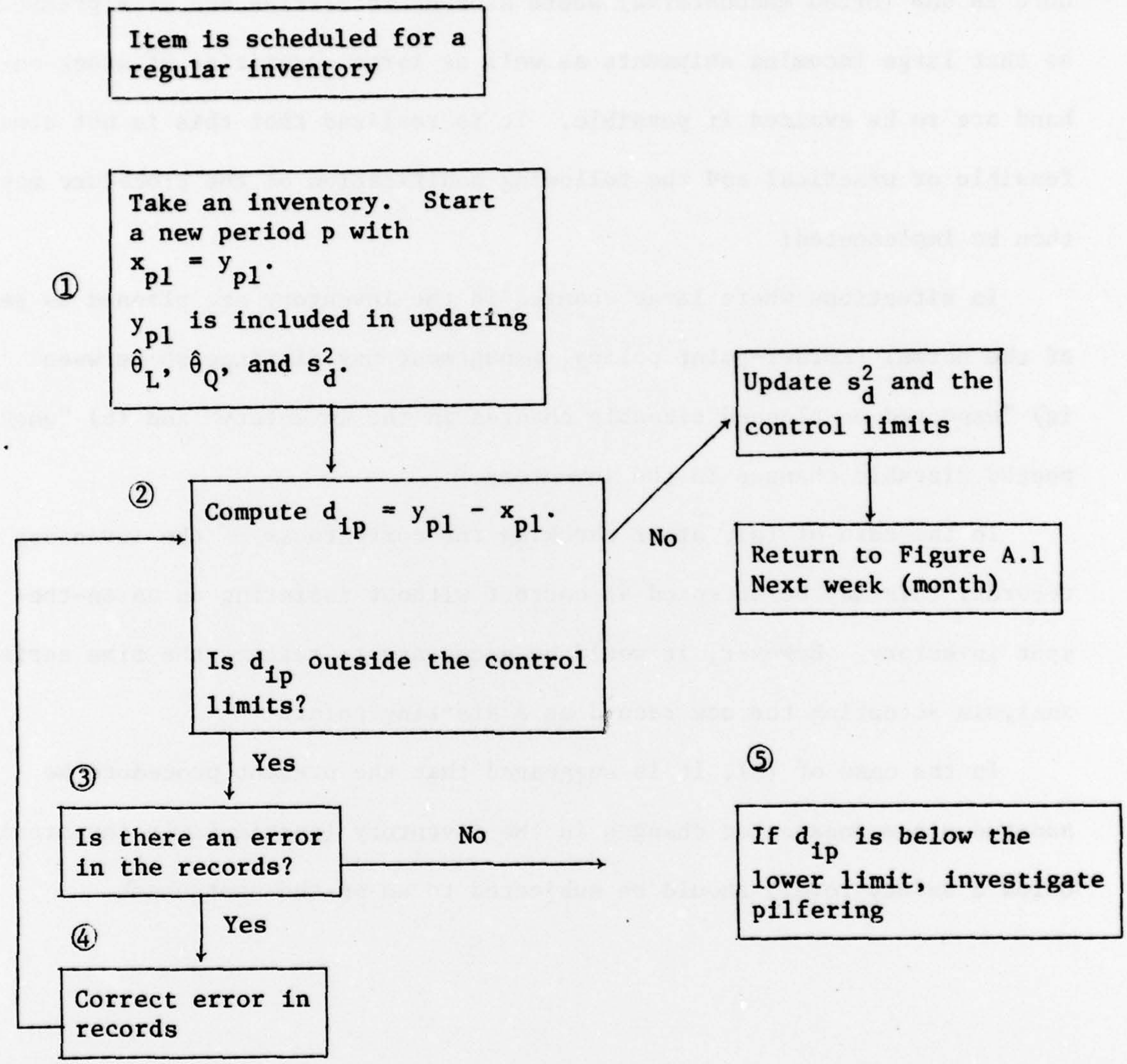
Figure (A.2)


Item is scheduled for a regular inventory
① Take an inventory. Start a new period p with $x_{p1} = y_{p1}$. y_{p1} is included in updating $\hat{\theta}_L$, $\hat{\theta}_Q$, and s_d^2.
② Compute $d_{ip} = y_{p1} - x_{p1}$.
Is d_{ip} outside the control limits?
No
Update s_d^2 and the control limits
Return to Figure A.1 Next week (month)
Yes
③ Is there an error in the records?
No
⑤ If d_{ip} is below the lower limit, investigate pilfering
Yes
④ Correct error in records

8. Pre-Planned Re-Orders and Cross-Leveling Actions

As mentioned in Section 1, the situation covered by the above procedure is one (often encountered) where storage facilities are at a premium so that large incoming shipments as well as large quantities of stock-on-hand are to be avoided if possible. It is realized that this is not always feasible or practical and the following modification of the procedure may then be implemented:

In situations where large changes in the inventory are planned as part of the normal reorder-point policy, management may distinguish between (a) "expected or planned sizeable changes in the inventory" and (b) "unexpected sizeable changes in the inventory."

In the case of (a), after checking the correctness of the inventory records, this may be accepted as correct without insisting on an on-the-spot inventory. However, it would be necessary to restart the time series analysis accepting the new record as a starting point.

In the case of (b), it is suggested that the present procedure be adopted since unexpected changes in the inventory (particularly inventories below a safety level) should be subjected to an on-the-spot check.

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